PVP 14

Code: EE4T1

II B.Tech - II Semester – Regular Examinations – May 2016

COMPLEX VARIABLES AND SPECIAL FUNCTIONS
(ELECTRICAL AND ELECTRONICS ENGINEERING)

Duration: 3 hours Max. Marks: 70

PART – A

Answer ***all*** the questions. All questions carry equal marks

11x 2 = 22 M

1)

a) Write the definition of analytic function.
b) If $\alpha + i\beta = tan^{-1}(x + iy)$ then find the real part α value.
c) Find the general value of $\log(-i)$.
d) Evaluate the complex integral $\int_C \frac{dz}{z-a}$ where C is the circle $|z - a| = r$.
e) Find the value of $\int_0^{z+i} (\bar{z})^2$ along the line $y = x/2$.
f) Write the definition of Isolated singularity of an analytic function.
g) Find the poles of $f(z) = \frac{z-3}{z^2+2z+5}$. Are these poles lie in the circle $|z + 1 - i| = 2$
h) Write the definition of conformal transformation.
i) Why the transformation $w = \frac{1}{z}$ is called inversion and reflection.

j) Express $j_3(x)$ in the use of $j_0(x)\&j_1(x)$.

k) Write the orthogonally condition for Legendre polynomials.

PART – B

Answer any ***THREE*** questions. All questions carry equal marks.

3 x 16 = 48 M

2)

a) If $w = \emptyset + i\Psi$ represents the complex potential for an electric find and $\Psi = x^2 - y^2 + \frac{x}{x^2+y^2}$ determine the function $\emptyset$. 8 M

b) If $f(z)$ is a regular function of z, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4\left|f'(z)\right|^2$ 8 M

3) a) Using Cauchy's integral formula evaluate the following integrals 8 M

(i) $\int_C \frac{e^{2z}}{(z-1)(z-2)}dz$, where C is the circle $|z| = 3$

(ii) $\int_C \frac{cos\pi z}{z^2-1}dz$ around a rectangle with vertices $2 \pm i, -2 \pm i$

(b)Expand $f(z) = \frac{1}{(z-1)(z-2)}$ in the region (i) $|z| < 1$

(ii)$1 < |z| < 2$ (iii) $|z| > 2$ 8 M

4)

a) Find the residue of $f(z) = \frac{z^3}{(z-1)^4(z-2)(z-3)}$ at its poles and hence evaluate $\int_C f(z)dz$ where C is the circle $|z| = 2.5$ 8 M

b) Evaluate $\int_0^{2\pi} \frac{cos3\theta}{5-4cos\theta} d\theta$ along a unit circle. 8 M

5)

a) Under the transformation $w = \frac{1}{z}$, find the image of (i) the circle $|z - 2i| = 2$ (ii) The straight line $y - x + 1 = 0$ 8 M

b) Find the bilinear transformation which maps the points $z = 1, i, -1$ into the w= $0, 1, \infty$. 8 M

6)

a) Prove that $xJ_n^{'}(x) = -nJ_n(x) + xJ_{n-1}(x)$ 8 M

b) Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre polynomials. 8 M